

# Algorithms for approximate string matching

Petro Protsyk (28 October 2016)



# Agenda

- About ZyLAB
- Overview
- Algorithms
  - Brute-force recursive Algorithm
  - Wagner and Fischer Algorithm
  - Algorithms based on Automaton
  - Bitap
- Q&A



# About ZyLAB

ZyLAB is a software company headquartered in Amsterdam.

In 1983 ZyLAB was the first company providing a **full-text** search program for MS-DOS called ZyINDEX

In 1991 ZyLAB released ZyIMAGE software bundle that included a **fuzzy string search** algorithm to overcome scanning and OCR errors.

Currently ZyLAB develops eDiscovery and Information Governance solutions for On Premises, SaaS and Cloud installations. ZyLAB continues to develop new versions of its proprietary full-text search engine.



# About ZyLAB

- We develop for Windows environments only, including Azure
- Software developed in C++, .Net, SQL
- Typical clients are from law enforcement, intelligence, fraud investigators, law firms, regulators etc.
  - FTC (Federal Trade Commission), est. >50 TB (peak 2TB per day)
  - US White House (all email of presidential administration), est. >20 TB
  - Dutch National Police, est. >20 TB



# ZyLAB Search Engine

## Full-text search engine

- Boolean and Proximity search operators
- Support for Wildcards, Regular Expressions and Fuzzy search
- Support for numeric and date searches
- Search in document fields
- Near duplicate search





# ZyLAB Search Engine

- Highly parallel indexing and searching
- Can index Terabytes of data and millions of documents
- Can be distributed
- Supports 100+ languages, Unicode characters



# Overview

Approximate string matching or **fuzzy search** is the technique of finding strings in text or dictionary that match given pattern approximately with respect to chosen **edit distance**.

The **edit distance** is the number of primitive operations necessary to convert the string into an exact match.

The usual primitive operations are: **insertion, deletion, substitution, transposition**.

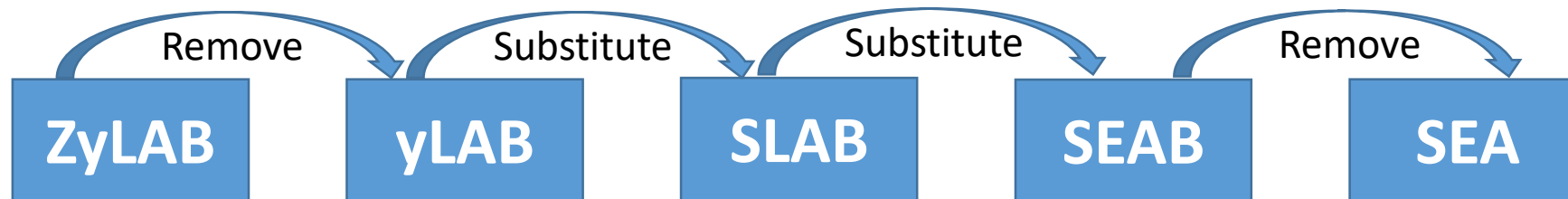
All operations have the same cost.

# Overview

**Levenshtein** distance between two words is the minimum number of single-character insertions, deletions or substitutions required to change one word into the other.

Named after Vladimir Levenshtein, who described this distance in 1965.

**Example:** Levenshtein distance between "ZyLAB" and "SEA" is 4:





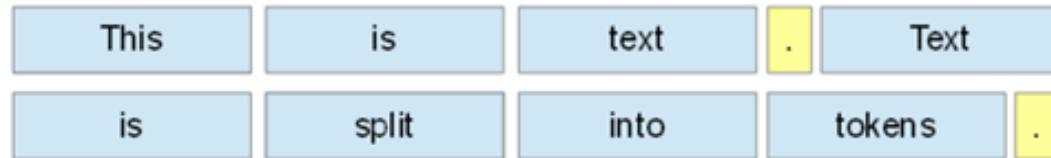


# Overview

**Applications:** The most common applications of approximate string matching are spell checking, syntax error correction, spam filtering, correction of OCR errors, full-text search.

# Full-text index

Text of the document

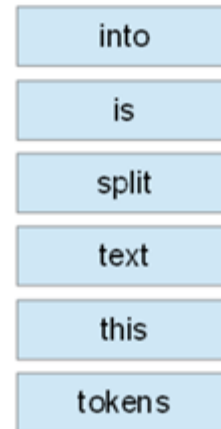


# Full-text index

## Text of the document

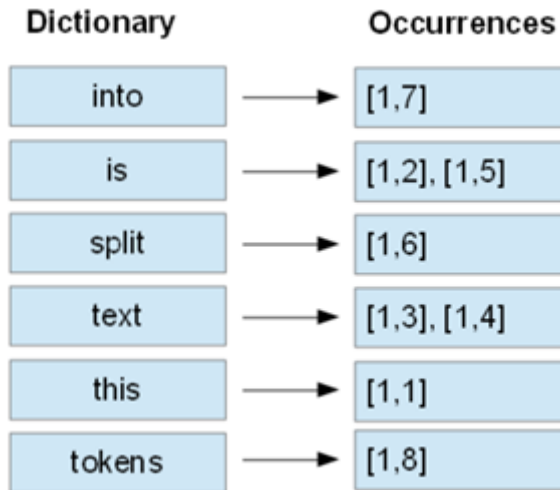
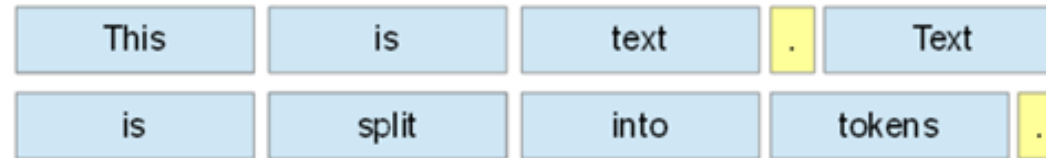


## Dictionary



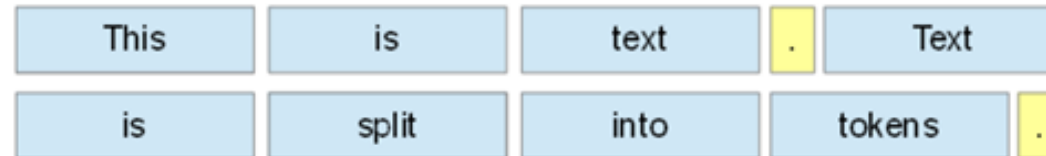
# Full-text index

## Text of the document



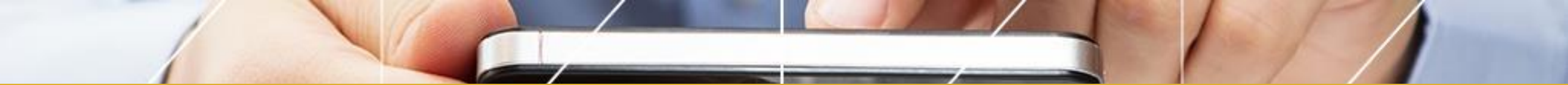
# Full-text index

Text of the document



Full-text Index

Dictionary		Occurrences
into	→	[1,7]
is	→	[1,2], [1,5]
split	→	[1,6]
text	→	[1,3], [1,4]
this	→	[1,1]
tokens	→	[1,8]



## Fuzzy query in ZyLAB search engine:

- All words with Levenshtein distance 2 or less: **word~2**
- All words with Levenshtein distance 2: **word~2 – {word~1}**





# Overview

**Problem:** Find terms in the dictionary that match the pattern approximately.

environment~2:

environment  
environments  
en**ve**ronment  
environment**al**

# Algorithms

Brute-force search  
(recursive, 1965)

Wagner and Fischer  
(Dynamic Programming, 1968-1974)

Levenshtein Automaton  
(Automata theory, 1992-2002)

Bitap algorithm  
(1964-1992)

# Algorithms

Mathematically, the Levenshtein distance between two strings  $a, b$  is given by the following formula  $lev_{a,b}(|a|, |b|)$ :

$$lev_{a,b}(i, j) = \begin{cases} \max(i, j) & \text{if } \min(i, j) = 0, \\ \min \begin{cases} lev_{a,b}(i-1, j) + 1 \\ lev_{a,b}(i, j-1) + 1 \\ lev_{a,b}(i-1, j-1) + 1_{(a_i \neq b_j)} \end{cases} & \text{otherwise.} \end{cases}$$

$lev_{a,b}(i, j)$  is the distance between the first  $i$  characters of  $a$ , and the first  $j$  characters of  $b$ .  $1_{(a_i \neq b_j)}$  is equal to 1 when  $a_i \neq b_j$  and 0 otherwise.

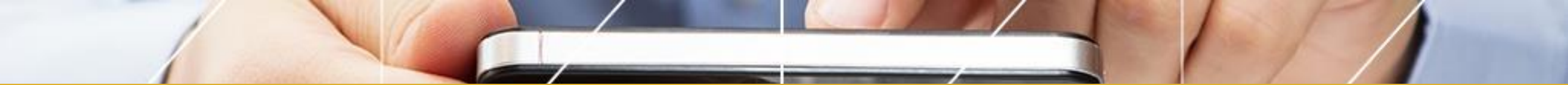
# Algorithms

$$\text{lev}_{a,b}(i, j) = \begin{cases} \max(i, j) & \text{if } \min(i, j) = 0, \\ \min \begin{cases} \text{lev}_{a,b}(i-1, j) + 1 \\ \text{lev}_{a,b}(i, j-1) + 1 \\ \text{lev}_{a,b}(i-1, j-1) + \mathbf{1}_{(a_i \neq b_j)} \end{cases} & \text{otherwise.} \end{cases}$$

**red** – deletion of character

**blue** – insertion of character

**green** – substitution or match



# Brute-force search

# Brute-force search

$$\text{lev}_{a,b}(i, j) = \begin{cases} \max(i, j) & \text{if } \min(i, j) = 0, \\ \min \begin{cases} \text{lev}_{a,b}(i-1, j) + 1 \\ \text{lev}_{a,b}(i, j-1) + 1 \\ \text{lev}_{a,b}(i-1, j-1) + \mathbf{1}_{(a_i \neq b_j)} \end{cases} & \text{otherwise.} \end{cases}$$



# Brute-force search

```
private static int CalculateRecursive(string a, string b, int m, int n)
{
    if (Math.Min(m, n) == 0)
    {
        return Math.Max(m, n);
    }

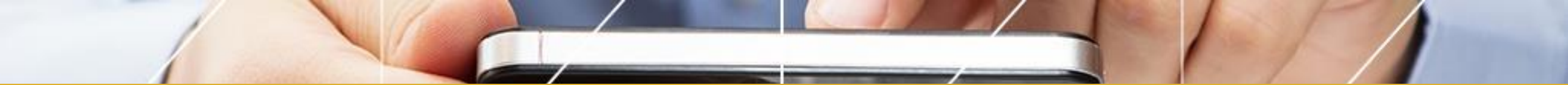
    var subCost = ((a[m - 1] == b[n - 1]) ? 0 : 1) + CalculateRecursive(a, b, m - 1, n - 1);
    var delCost = 1 + CalculateRecursive(a, b, m, n - 1);
    var insCost = 1 + CalculateRecursive(a, b, m - 1, n);

    return Math.Min(subCost, Math.Min(insCost, delCost));
}
```

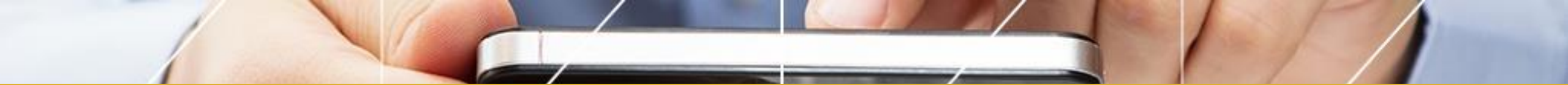


# Brute-force search

- 1) Easy to implement using  $lev_{a,b}(|a|, |b|)$  formula
- 2) Exponential complexity of  $|a|+|b|$
- 3) Not used in commercial applications



# Demo



# Wagner and Fischer Algorithm

# Wagner and Fischer Algorithm

## Idea of algorithm

Store Levenshtein distances between all prefixes of the first string and all prefixes of the second in the matrix.

To compute next cell, only values of three other neighbor cells are needed:

	<b>prefix(b,  b -1)</b>	<b>prefix(b,  b )</b>
<b>prefix(a,  a -1)</b>	<i>lev( a -1,  b -1)</i>	<i>lev( a -1,  b )</i>
<b>prefix(a,  a )</b>	<i>lev( a ,  b -1)</i>	<i>lev( a ,  b )</i>

# Wagner and Fischer Algorithm

**car**  $\longrightarrow$  **cdar**



# Wagner and Fischer Algorithm

**car**  $\longrightarrow$  **cdar**

					<b>c</b>
				<b>c</b>	<b>d</b>
		<b>c</b>	<b>d</b>	<b>a</b>	<b>r</b>
<b>-</b>	<b>0</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>

# Wagner and Fischer Algorithm

**car**  $\longrightarrow$  **cdar**

					<b>c</b>
				<b>c</b>	<b>d</b>
			<b>c</b>	<b>d</b>	<b>a</b>
		<b>-</b>	<b>c</b>	<b>d</b>	<b>a</b>
<b>-</b>	<b>0</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>
<b>c</b>	<b>1</b>	<b>0</b>	<b>1</b>	<b>2</b>	<b>3</b>

# Wagner and Fischer Algorithm

**car**  $\longrightarrow$  **cdar**

						<b>c</b>
					<b>c</b>	<b>d</b>
				<b>c</b>	<b>d</b>	<b>a</b>
		-	<b>c</b>	<b>d</b>	<b>a</b>	<b>r</b>
	-	<b>0</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>
	<b>c</b>	<b>1</b>	<b>0</b>	<b>1</b>	<b>2</b>	<b>3</b>
<b>c</b>	<b>a</b>	<b>2</b>				

# Wagner and Fischer Algorithm

**car**  $\longrightarrow$  **cdar**

						<b>c</b>
					<b>c</b>	<b>d</b>
				<b>c</b>	<b>d</b>	<b>a</b>
		-	<b>c</b>	<b>d</b>	<b>a</b>	<b>r</b>
	-	<b>0</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>
	<b>c</b>	<b>1</b>	<b>0</b>	<b>1</b>	<b>2</b>	<b>3</b>
<b>c</b>	<b>a</b>	<b>2</b>	<b>1</b>			

# Wagner and Fischer Algorithm

**car**  $\longrightarrow$  **cdar**

						<b>c</b>
					<b>c</b>	<b>d</b>
				<b>c</b>	<b>d</b>	<b>a</b>
		-	<b>c</b>	<b>d</b>	<b>a</b>	<b>r</b>
	-	<b>0</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>
	<b>c</b>	<b>1</b>	<b>0</b>	<b>1</b>	<b>2</b>	<b>3</b>
<b>c</b>	<b>a</b>	<b>2</b>	<b>1</b>	<b>1</b>		

# Wagner and Fischer Algorithm

**car**  $\longrightarrow$  **cdar**

						<b>c</b>
					<b>c</b>	<b>d</b>
				<b>c</b>	<b>d</b>	<b>a</b>
		-	<b>c</b>	<b>d</b>	<b>a</b>	<b>r</b>
	-	<b>0</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>
	<b>c</b>	<b>1</b>	<b>0</b>	<b>1</b>	<b>2</b>	<b>3</b>
<b>c</b>	<b>a</b>	<b>2</b>	<b>1</b>	<b>1</b>	<b>1</b>	



# Wagner and Fischer Algorithm

**car**  $\longrightarrow$  **cdar**

						<b>c</b>
					<b>c</b>	<b>d</b>
				<b>c</b>	<b>d</b>	<b>a</b>
		-	<b>c</b>	<b>d</b>	<b>a</b>	<b>r</b>
	-	<b>0</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>
	<b>c</b>	<b>1</b>	<b>0</b>	<b>1</b>	<b>2</b>	<b>3</b>
<b>c</b>	<b>a</b>	<b>2</b>	<b>1</b>	<b>1</b>	<b>1</b>	<b>2</b>

# Wagner and Fischer Algorithm

**car**  $\longrightarrow$  **cdar**

							<b>c</b>
						<b>c</b>	<b>d</b>
					<b>c</b>	<b>d</b>	<b>a</b>
			-	<b>c</b>	<b>d</b>	<b>a</b>	<b>r</b>
		-	<b>0</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>
		<b>c</b>	<b>1</b>	<b>0</b>	<b>1</b>	<b>2</b>	<b>3</b>
	<b>c</b>	<b>a</b>	<b>2</b>	<b>1</b>	<b>1</b>	<b>1</b>	<b>2</b>
<b>c</b>	<b>a</b>	<b>r</b>	<b>3</b>	<b>2</b>	<b>2</b>	<b>2</b>	<b>1</b>

# Wagner and Fischer Algorithm

**car**  $\longrightarrow$  **cdar**

								<b>c</b>				
								<b>c</b>				
							<b>c</b>	<b>d</b>				
						<b>c</b>	<b>d</b>	<b>a</b>				
					<b>c</b>	<b>d</b>	<b>a</b>	<b>r</b>				
					<b>-</b>	<b>c</b>	<b>d</b>	<b>a</b>	<b>r</b>			
					<b>-</b>	<b>0</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>		
					<b>c</b>	<b>1</b>	<b>0</b>	<b>1</b>	<b>2</b>	<b>3</b>		
					<b>c</b>	<b>a</b>	<b>2</b>	<b>1</b>	<b>1</b>	<b>2</b>		
					<b>c</b>	<b>a</b>	<b>r</b>	<b>3</b>	<b>2</b>	<b>2</b>	<b>2</b>	<b>1</b>

# Wagner and Fischer Algorithm

```
public static int CalculateDynamic(string a, string b)
{
    var d = new int[a.Length + 1, b.Length + 1];

    for (int i = 0; i <= a.Length; i++)
        d[i, 0] = i;
    for (int i = 0; i <= b.Length; i++)
        d[0, i] = i;

    for (int i = 0; i < a.Length; i++) {
        for (int j = 0; j < b.Length; j++) {
            if (a[i] == b[j]) {
                d[i + 1, j + 1] = d[i, j];
            } else {
                d[i + 1, j + 1] = 1 + Math.Min(d[i, j], Math.Min(d[i, j + 1], d[i + 1, j]));
            }
        }
    }

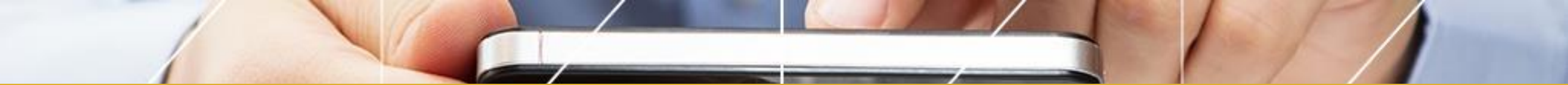
    return d[a.Length, b.Length];
}
```



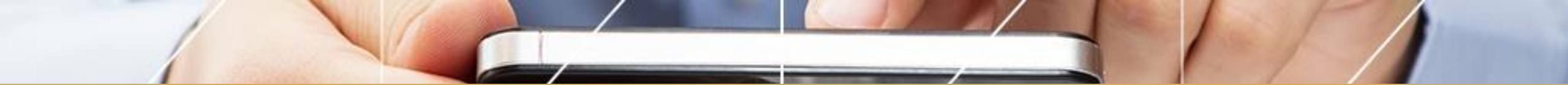
# Wagner and Fischer Algorithm

R Wagner, M Fischer The String-to-String Correction Problem (1974)

- 1) Easy to implement using  $lev_{a,b}(|a|, |b|)$  formula
- 2) Require  $O(|a|*|b|)$  steps, does not depend on input characters
- 3) Require  $O(\min(|a|, |b|))$  memory
- 4) Matching dictionary requires  $\sim |a|*\sum |t_i|$  steps



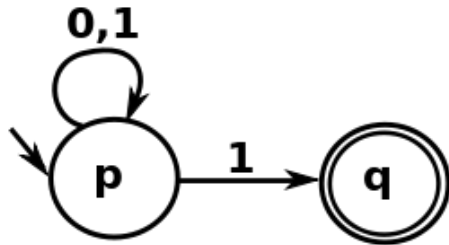
# Demo



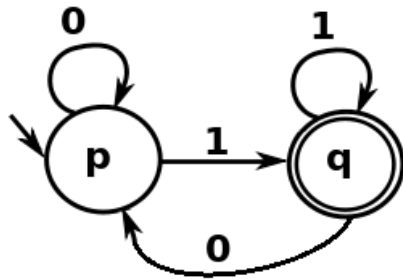
# Levenshtein Automaton

# Levenshtein Automaton

**Nondeterministic finite automaton** (NFA). States, alphabet, transitions, set of final states, initial state. For each symbol and state there can be multiple transitions.



**Deterministic finite automaton** (DFA). States, alphabet, transitions, set of final states, initial state. For each symbol and state there can be only one transition.



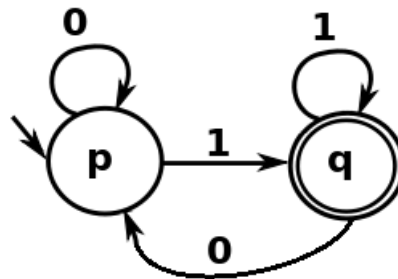


# Levenshtein Automaton

Automata can be used to recognize patterns in text

- 1) Begin from initial state
- 2) Follow transitions for each character in text
- 3) If resulting state is the final state – it is a match.

Example: 0 1 0 0 1



# Levenshtein Automaton

Matching dictionary with DFA is easy:

Difficult part

```
var dfa = CreateAutomaton(pattern, d);

foreach (var term in dictionary)
{
    var s = 0;
    foreach(var c in term)
    {
        s = dfa.Next(s, c);
    }
    if( dfa.IsFinal(s))
    {
        yield return term;
    }
}
```



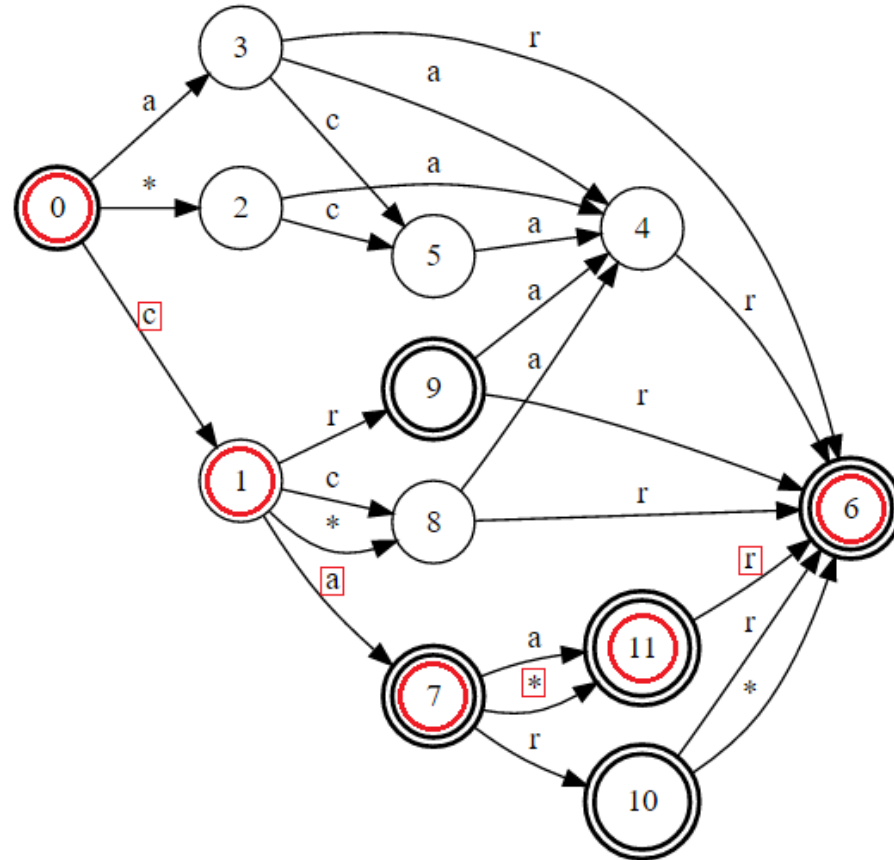
# Levenshtein Automaton

Levenshtein automaton for a string  $w$  and a number  $d$  is a finite state automaton that can recognize the set of all strings whose Levenshtein distance from  $w$  is at most  $d$

# Levenshtein Automaton

Automaton that recognizes: **car~1**

Matching **cadr**



A match require at most  $|b|$  steps



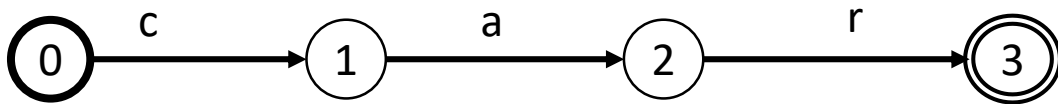
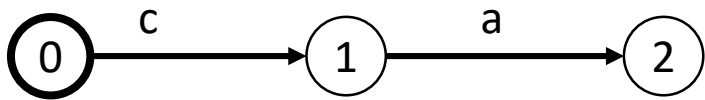
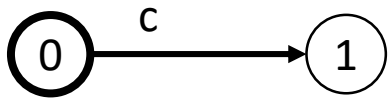
# Levenshtein Automaton

How to create Levenshtein Automaton for word **w**  
and a number **d**?

# Construction of NFA

Automaton to recognize **car**

①





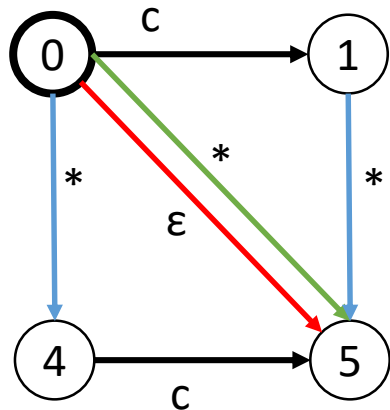
# Construction of NFA

Automaton to recognize **car~1**

①

# Construction of NFA

Automaton to recognize **car~1**



Recognize: **\_**, **a**, **c**, **ac**, **ca**

**blue** – insertion

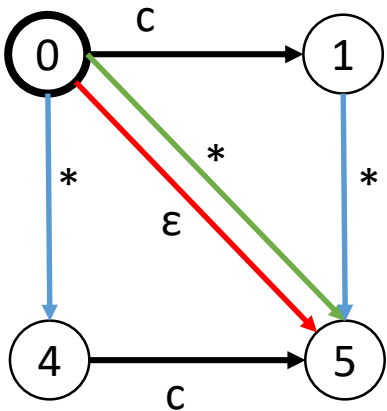
**red** – deletion

**green** - substitution



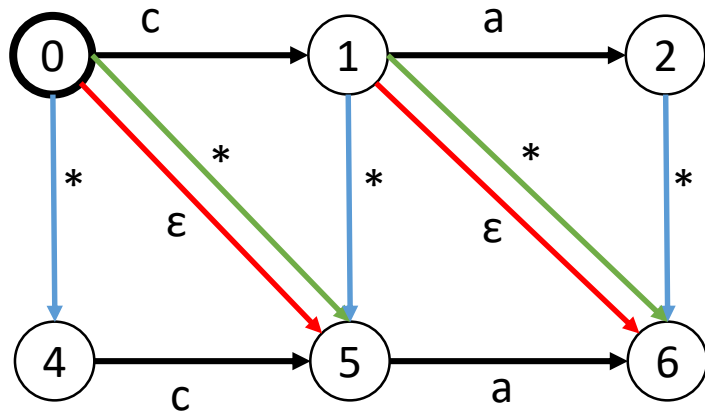
# Construction of NFA

Automaton to recognize **car~1**



# Construction of NFA

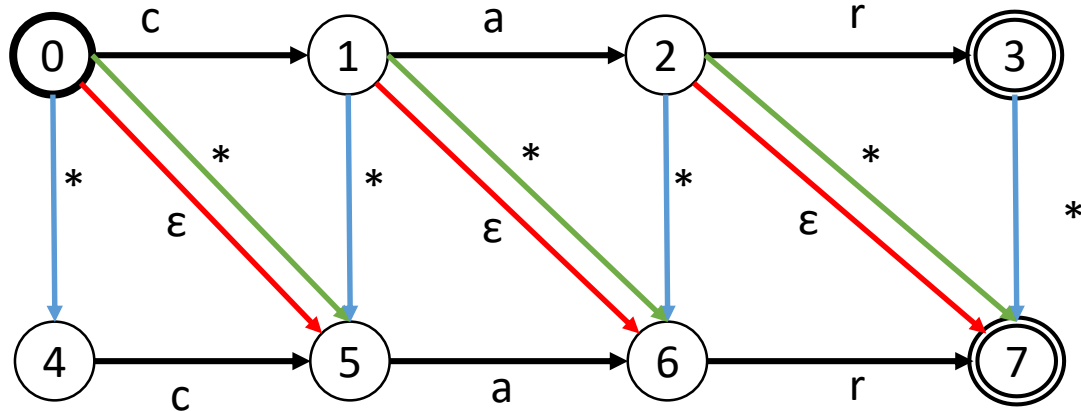
Automaton to recognize **car~1**



# Construction of NFA

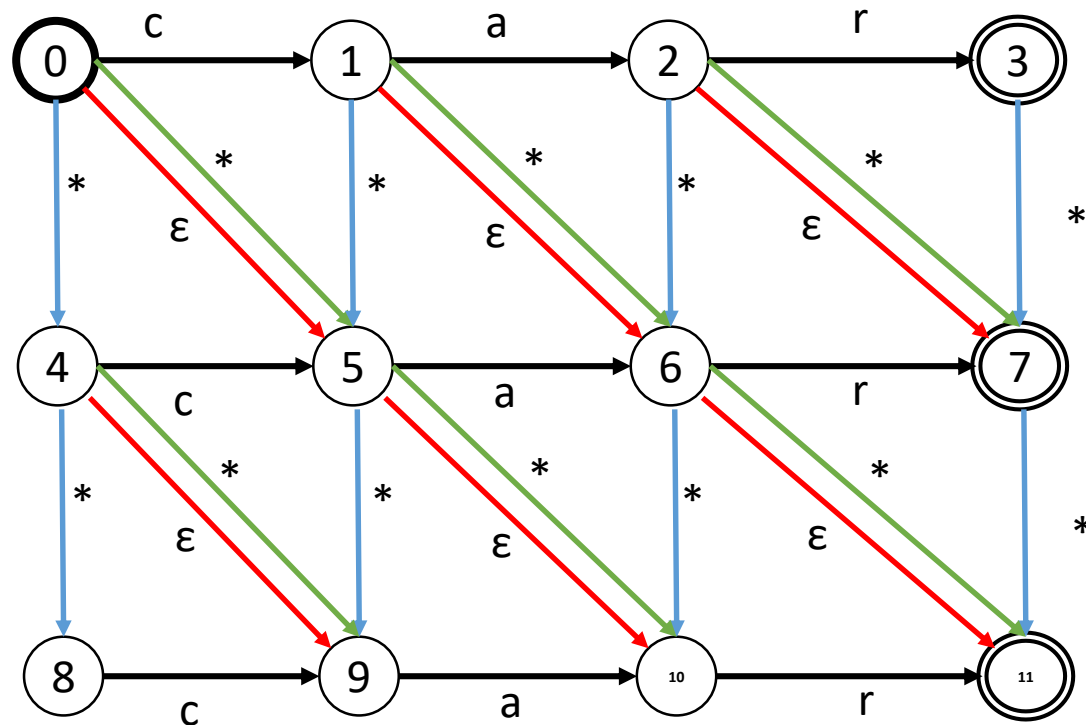
Automaton to recognize **car~1**

car~1



# Construction of NFA

car~2



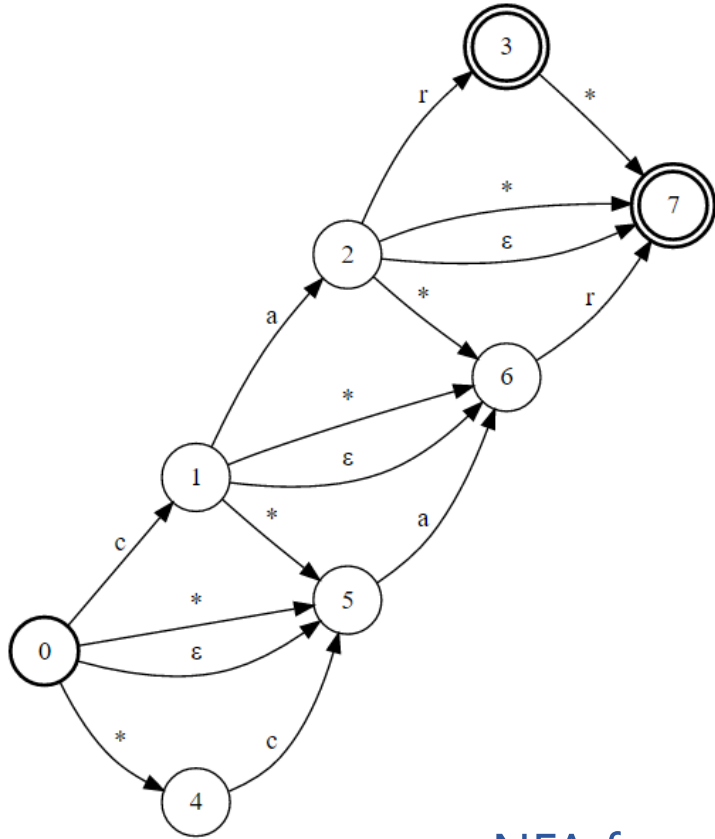
Each fuzzy degree adds one level



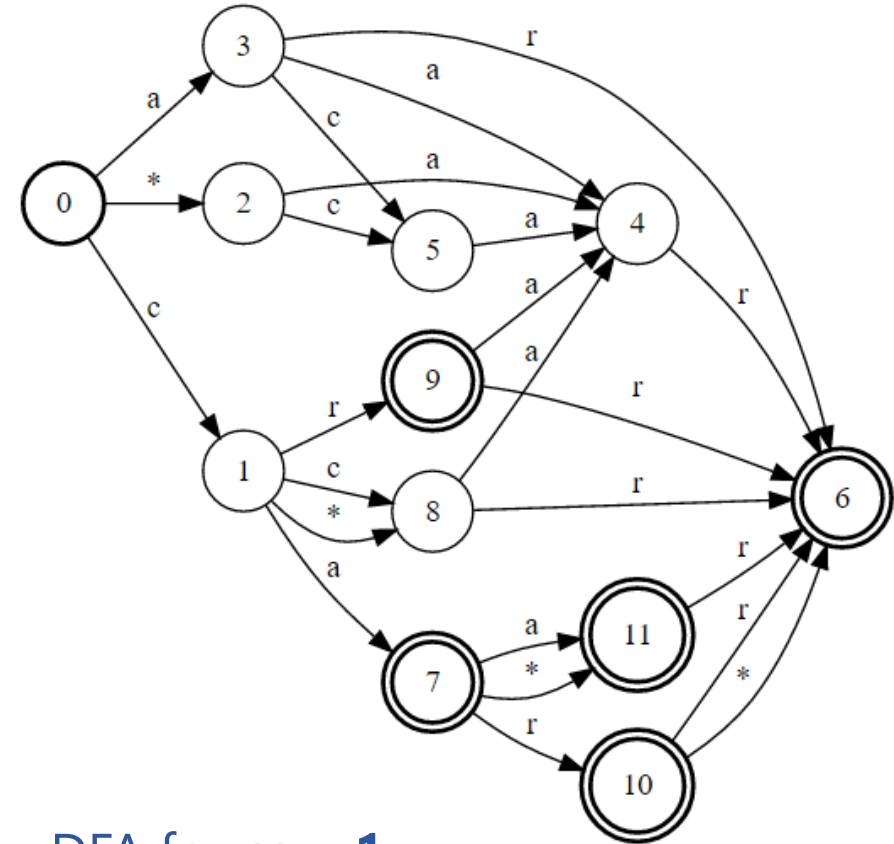
# Levenshtein Automaton

Every **NFA** can be converted to equivalent **DFA**

# Levenshtein Automaton



NFA for  $car^*1$

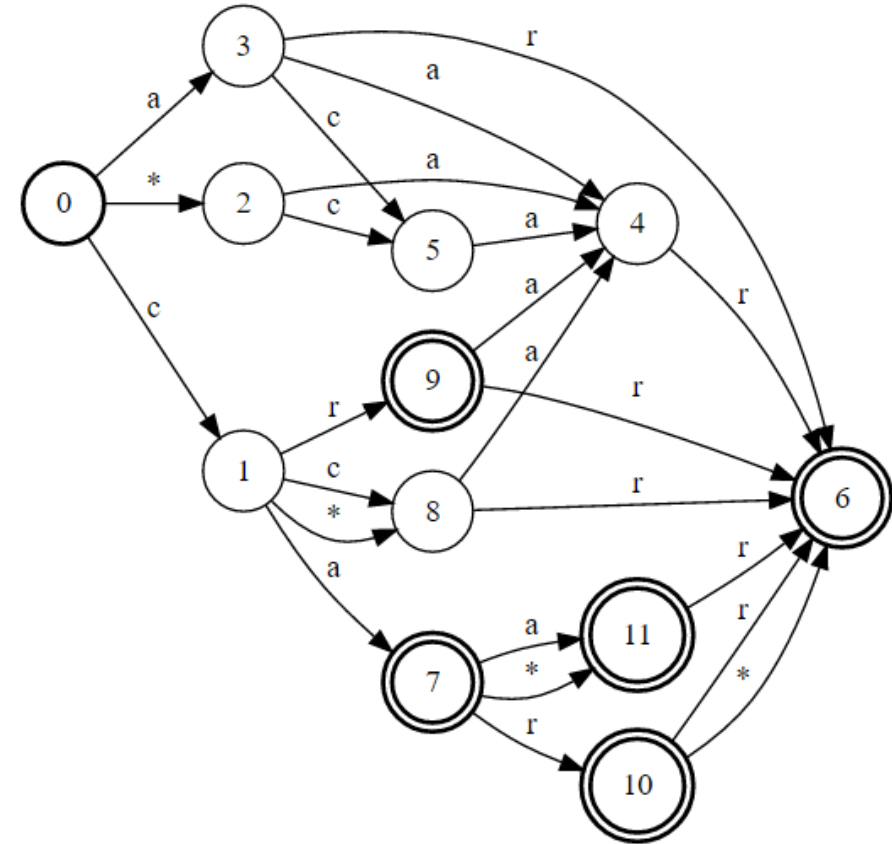


DFA for  $car^*1$

# Levenshtein Automaton

car~1

Linear construction



# Levenshtein Automaton

Matching dictionary with DFA is easy:

Difficult part

```
var dfa = CreateAutomaton(pattern, d);
```

```
foreach (var term in dictionary)
```

```
{
```

```
    var s = 0;
```

```
    foreach(var c in term)
```

```
    {
```

```
        s = dfa.Next(s, c);
```

```
    }
```

```
    if( dfa.IsFinal(s))
```

```
    {
```

```
        yield return term;
```

```
    }
```

```
}
```

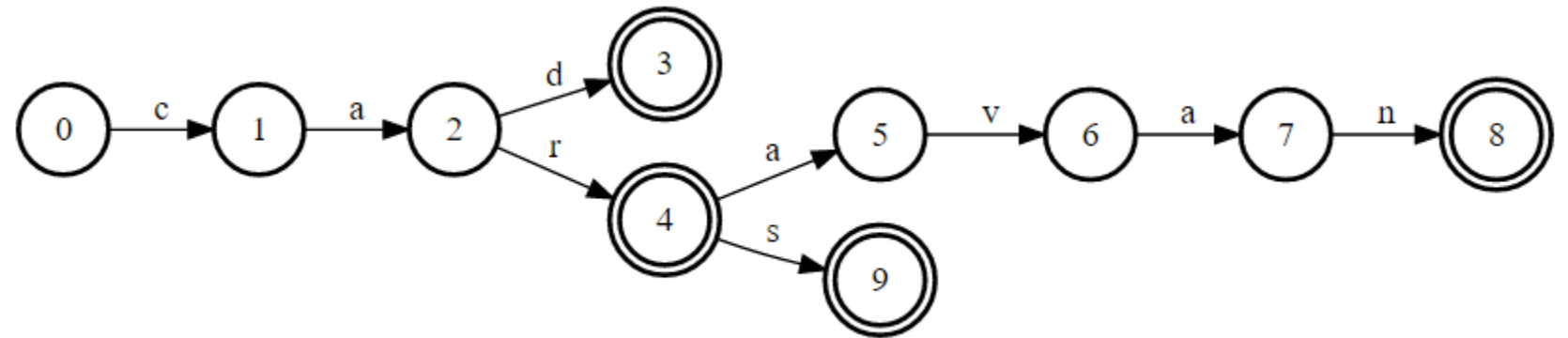
When current term and previous term share prefix, algorithm can start matching from the prefix



# Levenshtein Automaton

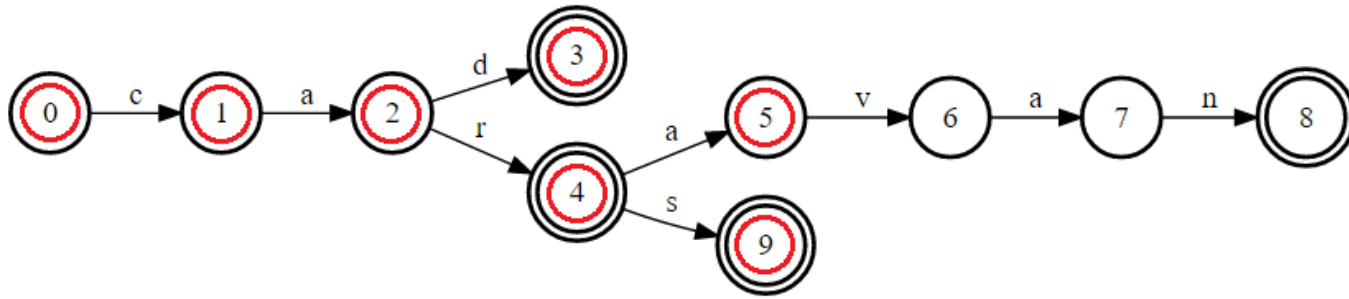
Matching a Prefix Tree based dictionaries

**cad**  
**car**  
**caravan**  
**cars**

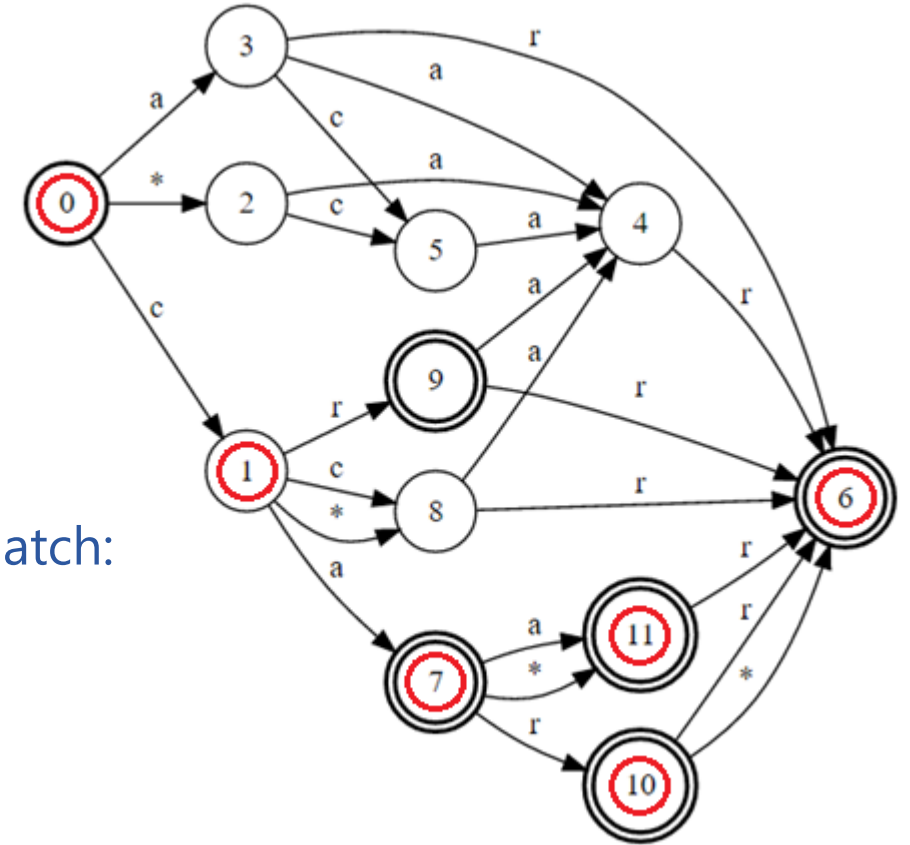


# Matching dictionary with Automaton

Dictionary DFA



Fuzzy term DFA



When both DFAs are in the final state (double circle) we have a match:

- cad
- car
- cars

Fuzzy DFA can be used to generate all possible terms that match it.



# Levenshtein Automaton

- 1) Difficult to implement
- 2) DFA construction might be slow for large degree **d**. In practice used when  **$d \in \{1,2,3,4\}$**
- 3) Matching dictionary requires less than  **$|a+d|*N$**  steps, where N number of terms in dictionary



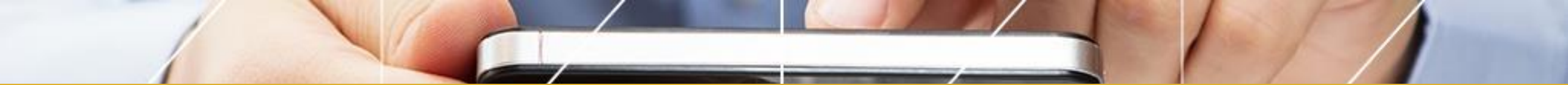
# Levenshtein Automaton

H. Bunke, A Fast Algorithm for Finding the Nearest Neighbor of a Word in a Dictionary (**1993**)

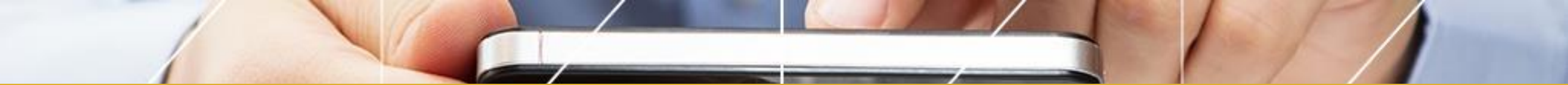
*Converting NFA to DFA using Rabin–Scott powerset construction (1959). if the NFA has  $n$  states, the resulting DFA may have up to  $2^n$  states*

K. Schulz , S Mihov, Fast String Correction with Levenshtein-Automata (**2002**)

*How to compute, for any fixed degree  $d$  and any input word  $W$ , a deterministic Levenshtein-automaton in time linear in the length of  $W$ .*



# Demo



# Bitap algorithm



# Bitap

**Bitap, Shift-Or, Shift-And** or **Baeza-Yates–Gonnet** algorithm

R. Baeza-Yates, G. Gonnet. A New Approach to Text Searching (1992)

Bálint Dömölki, An algorithm for syntactical analysis (1964)

# Bitap (Exact Match)

**Text:** ABABABC

**Pattern:** ABAB

	-
A	1
B	1
A	1
B	1

$R_0$

The update procedure for the  $R$  vector is:

$$R_0[i] = 1 \quad \text{for all } i = 1..m$$

$$R_{j+1}[1] = 0 \quad \text{when } p_1 = t_{j+1}$$

$$R_{j+1}[i] = 0 \quad \text{when } R_j[i-1] = 0 \text{ and } p_i = t_{j+1}$$



# Bitap (Exact Match)

**Text:** ABABABC

**Pattern:** ABAB

	-	A
A	1	0
B	1	1
A	1	1
B	1	1

$R_0$     $R_1$

The update procedure for the  $R$  vector is:

$$R_0[i] = 1 \quad \text{for all } i = 1..m$$

$$R_{j+1}[1] = 0 \quad \text{when } p_1 = t_{j+1}$$

$$R_{j+1}[i] = 0 \quad \text{when } R_j[i-1] = 0 \text{ and } p_i = t_{j+1}$$

# Bitap (Exact Match)

**Text:** ABABABC

**Pattern:** ABAB

	-	A	B
A	1	0	1
B	1	1	0
A	1	1	1
B	1	1	1

$R_0$     $R_1$     $R_2$

The update procedure for the  $R$  vector is:

$$R_0[i] = 1 \quad \text{for all } i = 1..m$$

$$R_{j+1}[1] = 0 \quad \text{when } p_1 = t_{j+1}$$

$$R_{j+1}[i] = 0 \quad \text{when } R_j[i-1] = 0 \text{ and } p_i = t_{j+1}$$

# Bitap (Exact Match)

**Text:** ABABABC

**Pattern:** ABAB

	-	A	B	A
A	1	0	1	0
B	1	1	0	1
A	1	1	1	0
B	1	1	1	1

$R_0$     $R_1$     $R_2$     $R_3$

The update procedure for the  $R$  vector is:

$$R_0[i] = 1 \quad \text{for all } i = 1..m$$

$$R_{j+1}[1] = 0 \quad \text{when } p_1 = t_{j+1}$$

$$R_{j+1}[i] = 0 \quad \text{when } R_j[i-1] = 0 \text{ and } p_i = t_{j+1}$$

# Bitap (Exact Match)

**Text:** ABABABC

**Pattern:** ABAB

	_	A	B	A	B
A	1	0	1	0	1
B	1	1	0	1	0
A	1	1	1	0	1
B	1	1	1	1	0

^

$R_0$     $R_1$     $R_2$     $R_3$     $R_4$

Match when value in the last row is 0

# Bitap (Exact Match)

**Text:** ABABABC

**Pattern:** ABAB

	-	A	B	A	B	A	B	C
A	1	0	1	0	1	0	1	1
B	1	1	0	1	0	1	0	1
A	1	1	1	0	1	0	1	1
B	1	1	1	1	0	1	0	1

  ^  ^

$R_0$    $R_1$    $R_2$    $R_3$    $R_4$    $R_5$    $R_6$    $R_7$



# Bitap

When pattern is small, vector R can be represented by unsigned integral value:

- uint – for patterns with up to 32 characters
- ulong – for patterns with up to 64 characters

# Bitap (Exact Match)

**Text:** ABABABC

**Pattern:** ABAB

**Preprocessing step:**

*Build T vectors for every character C in pattern:  
 $T[C][i] = 1$  if  $P_i=C$ , 0 otherwise.*

	T[A]	T[B]	T[C]
A	0	1	1
B	1	0	1
A	0	1	1
B	1	0	1

**The update procedure for the R vector:**

$$R_0 = 2^n - 1 \text{ (n number of bits in R)}$$
$$R_{j+1} = (R_j \ll 1) \mid T[t_j]$$

**Match when:**

$$R_j \& (\text{mask}) == 0$$

$$\text{Mask} = 1 \ll (|\text{Pattern}| - 1)$$

# Bitap (Exact Match)

**Text:** ABABABC

**Pattern:** ABAB

T[A]	T[B]	T[C]
0	1	1
1	0	1
0	1	1
1	0	1

	Bit	A
A	0	0
B	1	1
A	2	0
B	3	1
	4	0
	5	0
	6	0
	7	0
		R <sub>3</sub>



# Bitap (Exact Match)

**Text:** ABABABC

**Pattern:** ABAB

T[A]	T[B]	T[C]
0	1	1
1	0	1
0	1	1
1	0	1

	Bit	A	A<<1
A	0	0	0
B	1	1	0
A	2	0	1
B	3	1	0
	4	0	1
	5	0	0
	6	0	0
	7	0	0
		R <sub>3</sub>	

# Bitap (Exact Match)

**Text:** ABABABC

**Pattern:** ABAB

T[A]	T[B]	T[C]
0	1	1
1	0	1
0	1	1
1	0	1

	Bit	A	A<<1	T[B]	(A<<1) T[B]
A	0	0	0	1	1
B	1	1	0	0	0
A	2	0	1	1	1
B	3	1	0	0	0
	4	0	1	0	1
	5	0	0	0	0
	6	0	0	0	0
	7	0	0	0	0
		R <sub>3</sub>			

# Bitap (Exact Match)

**Text:** ABABABC

**Pattern:** ABAB

T[A]	T[B]	T[C]
0	1	1
1	0	1
0	1	1
1	0	1

	Bit	A	A<<1	T[B]	(A<<1) T[B]	B
A	0	0	0	1	1	1
B	1	1	0	0	0	0
A	2	0	1	1	1	1
B	3	1	0	0	0	0
	4	0	1	0	1	0
	5	0	0	0	0	0
	6	0	0	0	0	0
	7	0	0	0	0	0
		R <sub>3</sub>				R <sub>4</sub>

# Bitap

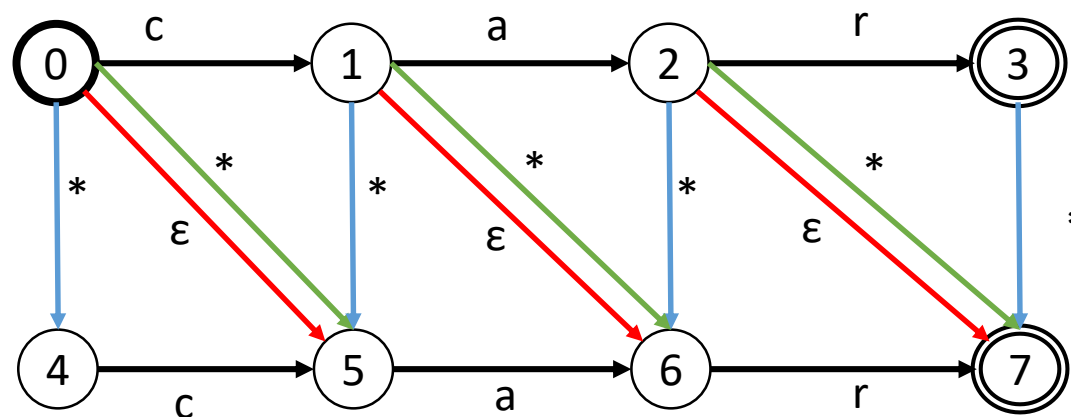
```
/* Initialize characteristic vectors T */
for (int i = 0; i < pattern.Length; ++i)
{
    T[pattern[i]] &= ~(1ul << i);
}

/* Initialize the bit array R. */
UInt64 R = ~(0ul);
UInt64 mask = 1ul << (pattern.Length - 1);
for (int i = 0; i < text.Length; ++i)
{
    R = (R << 1) | T[text[i]];
    if ((R & mask) == 0)
    {
        return (i - pattern.Length) + 1;
    }
}
return -1;
```

# Bitap

**Bitap** can be used for fuzzy matching with degree **d**.

The idea of fuzzy bitap is to simulate Levenshtein automaton (NFA) using bit-parallelism, so that each level of the automaton fits in a computer word (32, 64 or 128 states per level). For each new character, all transitions of NFA are simulated using bit operations among  $d+1$  computer words.





# Bitap

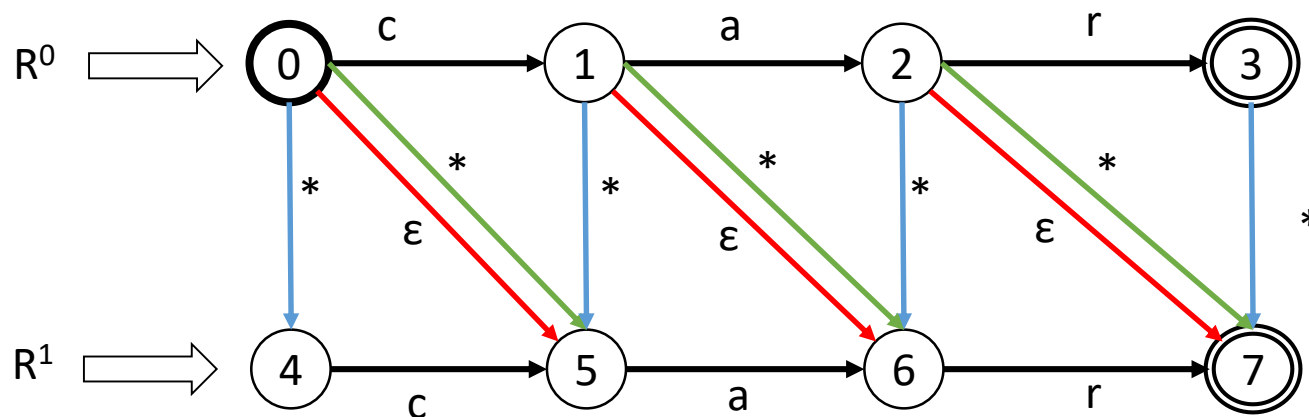
**Bitap** can be used for fuzzy matching with degree **d**.

- Instead of 1 vector, we need to keep additional **d+1** bit vectors:  $R^0_j, R^1_j, \dots, R^d_j$
- $R^d$  – tracks matching with at most **d** differences

# Bitap

**Bitap** can be used for fuzzy matching with degree **d**.

- Instead of 1 vector, we need to keep additional **d+1** bit vectors:  $R^0_j, R^1_j, \dots, R^d_j$
- $R^d$  – tracks matching with at most **d** differences



# Bitap

**Bitap** can be used for fuzzy matching with degree **d**.

- Instead of 1 vector, we need to keep additional **d+1** bit vectors:  $R^0_j, R^1_j, \dots, R^d_j$
- $R^d$  – tracks matching with at most **d** differences

Update procedure looks like this:

$$R^d_0 = 1 \dots 110 \dots (d \text{ times}) \dots 0 = (-1) \ll d$$

$$R^d_{j+1} = \boxed{R^{d-1}_j} \& \boxed{(R^{d-1}_{j+1} \ll 1)} \& \boxed{(R^{d-1}_j \ll 1) \& ((R^d_j \ll 1) | T[t_{j+1}])}$$

insertion

deletion

substitution

match





# Bitap

- 1) Easy to implement, might be difficult to understand
- 2) Memory required:  $O(|\mathbf{Pattern}| * |\mathbf{Alphabet}|)$
- 3) Complexity:  $O(|\mathbf{Text}|)$  when searching for all matches, or  $O(|\mathbf{Pattern}|)$  when searching for the first match
- 4) Efficient when **|Pattern|** is small: 8,32,64,128



Benchmark

**Demo**

# Benchmark

**Dataset:** Project Gutenberg, different languages, 2Gb of text

**Search Engine:** ZyLAB 6.6

**Dictionary:** 2178239 terms (2Mln)

**Query:** environment~2

	Time	Hits
Wagner and Fischer	1.585 sec	30771
Levenshtein automaton	<b>0.292 sec</b>	30771
Bitap	1.33 sec	30771



# References

- Levenshtein (1966). "Binary codes capable of correcting deletions, insertions, and reversals". Soviet Physics Doklady
- Wagner, Fischer, Michael (1974). "The String-to-String Correction Problem". Journal of the ACM
- Wu, Manber (1991). "Fast text searching with errors."
- Wu, Manber (1992). "Agrep - A Fast Approximate Pattern-Matching Tool"
- Baeza-Yates, Gonnet (1992). "A New Approach to Text Searching." Communications of the ACM, 35(10)
- Bunke (1993), A Fast Algorithm for Finding the Nearest Neighbor of a Word in a Dictionary
- Navarro, Gonzalo (2001). "A guided tour to approximate string matching". ACM Computing Surveys
- Schulz, Mihov (2002). "Fast String Correction with Levenshtein-Automata"